

Closed book. No calculators are to be used for this quiz.

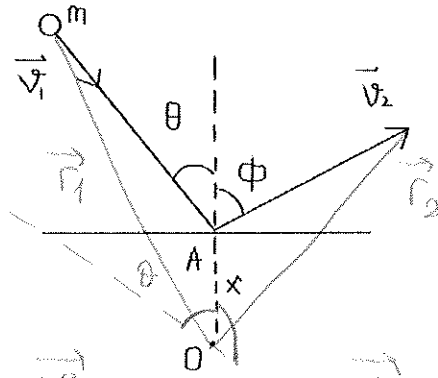
Quiz duration: 15 minutes

Name:

Student ID:

Signature:

A ball, of mass  $m$ , in a pool game is reflected from an edge of the pool table at point A (see the figure). Assume the table is well designed and friction is negligible. Show that the angle of incidence  $\theta$  is the same with the angle of reflection  $\phi$ . (Hint: Calculate the angular momentums of the ball, relative to some point O, just before and after the ball hits to the edge at A. Determine the speeds from the energy conservation.) (Expression for the moment of inertia I is not required. You can leave it as I in your answer.)



$$R = |r_1| = |r_2|$$

$$\vec{L}_1 = \vec{r}_1 \times m_1 \vec{v}_1$$

$$\vec{L}_2 = \vec{r}_2 \times m \vec{v}_2$$

$$L_1 = m_1 R v_1 \sin(\pi - \theta)$$

$\theta$

$$L_2 = m_1 R v_2 \sin \phi$$

$$E_1 = \frac{1}{2} m v_1^2 \quad E_2 = \frac{1}{2} m v_2^2 \quad m_2 = m \text{ and } E_1 = E_2$$

$v_1 = v_2$

$$L_1 = L_2$$

~~$$m_1 R v_1 \sin \theta = m_1 R v_1 \sin \phi$$~~

$$\sin \theta = \sin \phi$$

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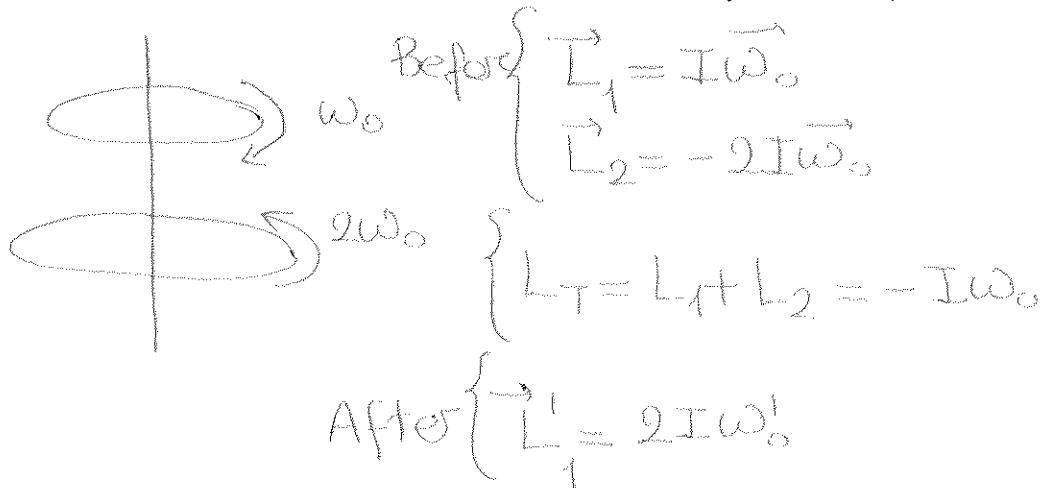
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Two identical disks with moment of inertia  $I$  are rotating on the same vertical shaft, one at the top, at angular speed  $\omega_0$  in clockwise direction, while the other at  $2\omega_0$  in the counterclockwise direction. The top disk is suddenly released and falls on the bottom one and they stick together. What is their common angular speed? (Expression for the moment of inertia  $I$  is not required. You can leave it as  $I$  in your answer.)



$$L'_1 = L_T$$

$$2I\omega'_0 = -I\omega_0$$

$$\boxed{\omega'_0 = -\frac{\omega_0}{2}}$$

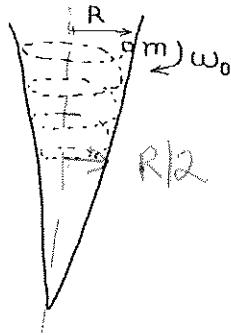
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A particle of mass  $m$  is thrown into a conical bowl with an angular speed  $\omega_0$  (for the rotation about the symmetry axis of the cone). It spirals (rotation about the symmetry axis of the cone while translationally moving downward to the bottom of the cone) down on the inner surface of the cone as shown in the figure. Initially the particle is at radius  $R$  away from the symmetry axis of the cone. Calculate the angular speed  $\omega$  (for the rotation about the symmetry axis of the cone), in terms of the initial angular speed, when the particle is at radius  $R/2$ .



Angular momentum  
 $L_i = L_f$  conserved!

$$L_i = I_i \omega_i = mR^2 \omega_i$$

$$L_f = I_f \omega_f = m\left(\frac{R}{2}\right)^2 \omega_f$$

$$mR^2 \omega_i = m \frac{R^2}{4} \omega_f$$

$$\omega_f = 4\omega_i \quad \omega_i = \omega_0$$

$$\boxed{\omega_f = 4\omega_0}$$

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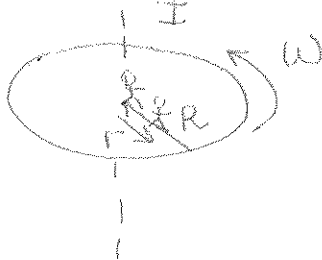
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A turntable with moment of inertia  $I$  makes one revolution in every  $T$  seconds. A man of mass  $m$  standing at the center of the turntable runs out along a radius fixed in the turntable. What is the angular velocity when the man is at a distance  $r$  away from the center? (Expression for the moment of inertia  $I$  is not required. You can leave it as  $I$  in your answer.)



Revolution =  $T$

$\omega$  of man?

$$L_0 = I\omega \quad (\text{man at center})$$

$$L_1 = (I + \underbrace{mr^2}_{\text{man at } r})\omega'$$

$$L_0 = L_1 \quad I\omega = I\omega' + mr^2\omega'$$

$$\omega = \frac{2\pi}{T} \quad (\text{initial})$$

$$\frac{2\pi I}{T} = I\omega' + mr^2\omega'$$

$$\omega'(I + mr^2) = \frac{2\pi I}{T}$$

$$\boxed{\omega' = \frac{2\pi I}{T(I + mr^2)}}$$

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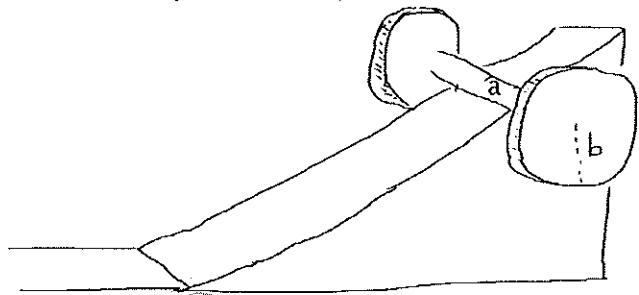
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A dumbbell of mass  $M$  and moment of inertia  $I$  is rolling down an incline, without slipping, as shown in the figure. Calculate the angular momentum of the dumbbell just before and after it touches the ground. Assume friction is small enough so that the system is isolated. (Expression for the moment of inertia  $I$  is not required. You can leave it as  $I$  in your answer.)



① Before landing

$$R = a$$

② After landing

$$R = b$$

⑤

$$E_i = mgh \quad \textcircled{3}$$

$$L = I\omega \quad \textcircled{1}$$

$$E_{\text{before}} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 = E_i$$

$$v = \omega a$$

$$= \frac{1}{2} m \omega_b^2 a^2 + \frac{1}{2} I \omega_b^2$$

$$E_{\text{after}} = \frac{1}{2} m \omega_a^2 b^2 + \frac{1}{2} I \omega_a^2 = E_i$$

$$L_a = I \omega_a \quad (\text{after})$$

$$L_b = I \omega_b \quad (\text{before})$$

$$L_b \Rightarrow 2E_i = \omega_b^2 (ma^2 + I)$$

$$I \omega_b = I \sqrt{\frac{2E_i}{ma^2 + I}} = L_b$$

$$L_a \Rightarrow 2E_i = \omega_a^2 (mb^2 + I)$$

$$I \omega_a = I \sqrt{\frac{2E_i}{mb^2 + I}} = L_a$$

$$\frac{L_{\text{before}}}{L_{\text{after}}} = \frac{\sqrt{2E_i} \sqrt{mb^2 + I}}{\sqrt{ma^2 + I} \sqrt{2E_i}} = \sqrt{\frac{mb^2 + I}{ma^2 + I}}$$